Abstract: This article focuses on the experiences of two researchers, Wolff-Michael ROTH and Luis RADFORD, using cultural-historical activity theory in mathematics education. The aim is to provide insights into the ways these researchers see and engage with activity theory, how they have come to adopt and expand it, and some of the challenges and concerns that they have had using it. These questions are not usually addressed within typical scientific papers. Yet, they are important for understanding both the dynamics of research and the practical use of cultural-historical activity theory. Since the format of research report papers is not necessarily well suited to convey personal experiences and thinking, the present article takes the form of a conversation, which provides an effective vehicle for exploring and articulating these matters. This provides a basis for understanding more deeply the underlying assumptions of this theory; its dynamics and how it is applied in research of mathematics practice, thinking, and learning; and insights into the manner in which experienced researchers grapple with the theoretical dimensions of their research.

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1. Background to Activity Theory

Lionel: Activity theory or cultural-historical activity theory (CHAT) is a cross-disciplinary framework for studying how humans purposefully transform natural and social reality, including themselves, as an ongoing culturally and historically situated, materially and socially mediated process. Rooted in the dialectical psychology of VYGOTSKY (e.g., 1978) and LEONT’EV (e.g., 1978, 1981), this perspective transcends traditional dichotomies of micro and macro, internal and external, mental and material, individual and social, thought and action, quantitative and qualitative, observation and intervention, as well as agency and structure by integrating three perspectives: the objective, the ecological, and the socio-cultural (ENGESTRÖM, 1993). An essential feature of activity theory is the recognition of subject, object, community, material and semiotic tools, and other
features of cultural practice as constitutive moments of activity—the irreducible, minimum unit of analysis. [1]

CHAT offers a radical departure from the dualistic views of thinking and being that pervade most other theories of human interaction, learning, and development. As a result, understanding the epistemological and ontological commitments of activity theory along with their implications for interpreting human activity can be a significant challenge for newcomers. Compounding this are the different takes on activity theory in the research literature reflecting both the wide range of problems and contexts where this theory is applied as well as varying degrees of consistency of existent research with its foundational principles. [2]

The present article provides insights to the ways that two accomplished activity theory researchers, Michael ROTH and Luis RADFORD, have come to adopt and apply this perspective in their own research related to mathematics practice, thinking, and learning.¹ A semi-structured interview with these researchers was conducted, video recorded, and transcribed verbatim. Each of the three authors then shared in the editing of the transcript for clarity. The questions that guided this effort were as follows: What motivated the researchers to adopt a CHAT perspective for their own work? What were the dynamics of their becoming acquainted with and adopting a CHAT perspective? What do they see as essential features of CHAT? And, what challenges and concerns do they have working with it? In short, this article provides insights to these researchers' own activity of engaging with CHAT. [3]

2. Background to the Conversation

The conversation with ROTH and RADFORD that is documented here took place on February 24, 2010 when ROTH was visiting RADFORD's Cultural Semiotics and Mathematical Thinking Lab at Laurentian University, in Sudbury, Ontario. [4]

ROTH has served as the editor of the leading journal for activity theoretical research, Mind, Culture, and Activity, and has contributed extensively to the elaboration of cultural-historical activity theory including a recent review of the literature on the topic (ROTH & LEE, 2007). He has published "The Eternal Return: Reproduction and Change in Complex Activity Systems—The Case of Salmon Enhancement" (ROTH, LEE & BOYER, 2008) and "Participation, Learning, and Identity: Dialectical Perspectives" (ROTH, HWANG, LEE & GOULART, 2005) that are included in a special book series that focuses on republishing the work of the founders of activity theory augmented by recent work drawing on the materialist-dialectical framework. RADFORD's (2008) ground breaking theory of knowledge objectification provides an elaboration and extension of activity theory specific to mathematics teaching and learning, based on his extensive experience working in elementary and secondary mathematics classrooms. He has published "Communication et apprentissage" (RADFORD &

¹ While ROTH's research spans both mathematics and science education, the focus in this article is on mathematics education. The discussion of theory and methodology applies, nevertheless, within both of these research domains and others.
This conversation was organized and moderated by Lionel LaCROIX who recently completed his doctoral dissertation (2010) using RADFORD's theory of knowledge objectification as well as ENGESTRÖM's interpretation of CHAT to analyze mathematics practice and learning. The questions that he puts to ROTH and RADFORD speak to issues that he grappled with in the process of coming to understand and adopt a CHAT perspective in his own work.

3. The Conversation: Coming to Activity Theory

Lionel: To start, Luis, what drew you to activity theory as a basis for your own thinking and research in mathematics education?

Luis: Well, in order to answer this question, I have to come back to the 1980s, the period when I was doing my Ph.D. in France. A common denominator in my work is that I have been trying to understand mathematical thinking. I was trained in a program where the Piagetian component was very strong, so thinking was investigated within the framework of Piagetian epistemology. I had a sense—but it was very vague—that there were different ways to think mathematically about the world. My contact with BROWER's (1952) intuitionism and multimodal logic was very revealing. However, apart from the fascinating philosophical reasons that underpin the various schools of mathematical logic and account for their differences, I couldn't find support in the psychological literature at the time to deepen this idea. I have to say that when I was doing my Ph.D., VYGOTSKY was only being translated from Russian into French and cross-cultural psychology was not something that was very strong within the French math education community, even though they have one of the most influential schools of sociology (i.e., the DURKHEIM school). So, it was just a sense that I had but I couldn't articulate it in a more precise way.

As I moved from culture to culture and from country to country, it became clear for me that cultures influence the way we think about the world in a way that was not acknowledged sufficiently at the time. In 1991, after my Ph.D. and spending six more years in Guatemala—my country of origin, I moved to Montreal where I started working on questions about the history of mathematics. It became clear for me that the cultural-historical forms of thinking that I could access through historical documents were very different from one period to another. At the time historians were explaining development mainly from the modern point of view—that some of them were calling the problem of presentism. But I lacked a
coherent theoretical framework to conceptualize mathematical thinking from an anthropological perspective. And I found, in what we now call activity theory, some tools for understanding the different forms of thinking that I could see through the history of mathematical ideas. My first serious attempt was a paper that appeared in "For the Learning of Mathematics" (RADFORD, 1997). This paper was an important step for me in the manner that I eventually conceptualized the relationship between thought and culture. [9]

I remember that at the end of the 1980s, perhaps 1988 or '89, I went to Mexico and bought a Spanish translation of "Thought and Language" ("Pensamiento y lenguaje", VYGOTSKY, n.d.). It impressed me deeply. Yet I was not able to formulate my research problems at that time within a Vygotskian perspective. It took me almost ten years to be able to state my research problems in terms of Vygotskian concepts. But what led me to activity theory was this sense that cultures were much more influential than was acknowledged at the time. The testimonies that I was seeing in the history of mathematics were intimating that the ways of thinking that were behind the mathematics of different historical periods were completely different. This made me think that there was something missing in the ways we were talking about the development of mathematical thinking in history and in students. [10]


Michael: For me it was an experience of emergence in the sense that during my dissertation I was, like Luis, working with Piagetian concepts as well as neo-Piagetian concepts that integrated information processing and short-term memory into Piaget's stage theory, and there seemed to be a short step from there to radical constructivism. But, as soon as I went from the university, from my dissertation, back into the classroom where I had always emphasized group work, all the work of individual constructivism didn't seem to make sense anymore as I was attempting to come to grips with the conversations that students have and a sense that they were understanding each other even though their reasons weren't fully articulated. There was something going on at a collective level, at a group level, that I could not capture with the theoretical frameworks I had. Then I began to think in terms of collective, of group, of the social, and then, over time, I learned about discursive psychology that focuses on language, language as something that is shared. [12]

It was during that time that I read ENGESTRÖM (e.g., 1993) who is one interpreter of CHAT, especially as developed by LEONT'EV (1978). I was also familiar with the kind of stuff that he and Mike COLE had written (COLE & ENGESTRÖM, 1993) without that work affecting what I was doing. But—as in many other instances where I have tried out a theoretical framework because the data that I was working with didn't make any sense—I was looking for other ways to frame what I was confronted with. [13]

In the same way, I was confronted again in the late 1990s with trying to understand how new teachers become when they teach with other people in
research on co-teaching. We were working with a mix of students in classrooms, from the poorest areas of Philadelphia, who were destined to failure within these schools and with their teachers. We tried to train teachers to work in these schools and I had, with Ken TOBIN, decided to do some work together on the various dimensions of this project. I was already very familiar with practice theory by BOURDIEU (1980) and I attempted to understand the multilayered events that were present within this situation. We had new teachers there—individuals or people who were in their professional year—and we sent them out into school classrooms to teach with teachers there on a daily basis. And at the same time the university people, like myself; when we were in the classrooms doing research we had to teach—nobody was allowed in the classroom to observe without actually contributing to the teaching. But then after a lesson, after a typical 90-minute lesson, we would go to another room and then ask the question, "So what has happened just now?" We were looking at our own activity, at what we had done and we had started to include students. We met together: the beginning teachers, teachers in training, expert teachers, university professors, and the university methods instructor. We all sat around a table together with a couple of students and asked, "So what happened? How can we make it better?" [14] It was this question, "How can we make it better?" that sort of drew me to HOLZKAMP (1983), who was also influenced by and further developed the activity-theoretical work of LEONT’EV. It became clear to me that there were multiple activities here. One was when we were teaching and another when we were looking at the teaching together. There are two forms of consciousness then: they’re very different because in the one instance we’re teaching, in the other instance we’re talking about teaching. And all of a sudden it became clear to me that activity theory provided a useful framework. I began to play with triangles, you know having read ENGESTRÖM, I played with triangles and I struggled and struggled and struggled. But I persevered because we had this project, I was writing four articles and then later on four chapters—understanding teaching using activity theory, understanding researching teaching doing activity theory, understanding evaluation of teaching doing activity theory, and understanding content matter learning of the researchers. I was playing around a lot with triangles and it was at that moment that there was a confluence of my reading of HOLZKAMP and my reading of ENGESTRÖM: they somehow came together. And then I sought to deepen my understanding of activity theory. I re-read LEONT’EV (1982) and IL’ENKOV (e.g., 1977); and then went back to MARX and ENGELS (e.g., 1968 [1890]) because LEONT’EV fundamentally—throughout his book—implicates his intellectual heritage with MARX. In fact the first German editions of LEONT’EV’s book "Tätigkeit, Bewusstsein, Persönlichkeit" [Activity, Consciousness and Personality] (1982) had the first two chapters chopped because they were all about MARX. IL’ENKOV also acknowledges this intellectual heritage. So for me, activity theory became a way of articulating what interested me in MARX and ENGELS’ "Eleventh Thesis on Feuerbach" (1978 [1924])—that philosophers had tried to understand the world and our purpose as educators was to transform it. And in activity theory I saw a way of articulating this transformatory aspect that was the focus of our research. We didn't just want

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to understand the classrooms we were working in: we wanted to transform them. But not we transform; we wanted to assist the agents of the classroom. We wanted teachers and students to be able to transform, to work together, to transform independent of our being there. But of course it took our being there, perhaps as an impetus, to get it started, to facilitate their interactions. [15]

Luis: This is, I think, very important from an activity theory perspective, that we are not merely observers of what's going on in the world. We are trying to transform it. It's a completely different philosophical perspective of practice and a completely different theoretical approach to practice, not as observers but as implicated in some way in what is going on there, hoping that the transformations that have been produced through our own participation along with the students and the teachers, participation in which we have also been transformed as researchers, may continue over and over. [16]

3.1 Essential features of cultural-historical activity theory

Lionel: Michael, what do you see as the essential core idea or ideas at the heart of activity theory? What is essential for someone coming to activity theory from a different perspective to be able to understand the world from an activity-theoretical perspective? [17]

Michael: Well there are two things. One, we're dealing with a name. We have activity theory or, as I try to emphasize, cultural-historical activity theory. These important dimensions are not necessarily realized when people refer to using the theory. It's in the name. The important things are in the name: culture, history, and activity. After I struggled for a few years to develop a sense that I am understanding what is going on, I began to say to my graduate students, "Don't look for subjects or tools or objects, and so on. Look for something that is an event—activity. Something has to happen." So we're not talking about activity in the abstract, about thinking in the abstract. We're interested in understanding something that's going on. It is an event, so we have to think in terms of eventness, of change, that's one. Whatever we observe, we cannot understand independent of the culture within which this occurs, with respect to where intelligibility is presupposed. Humans tend to act in ways that they presuppose to be intelligible so that our individual actions never are just radically our own. They're inherently intelligible which means they're already realizing possibility of the other as well. [18]

Culture and history. History introduces a second temporal dimension. It's not just that activity is temporal, but the activity itself is in an historical context. So what we are observing today is different from what we might have observed in the same room, with the same teacher, teaching the same subject or from the same book, it will be different from what we have observed ten years earlier. So we need to take into account that any kind of human-human interactions that we look at is contextualized by activity. What are they producing? What is this all about? Culture and history. And, of course, we have to unpack what each of these means. There is a double temporal component, the local temporality as well as
the historical and the cultural embeddedness. And, that is a radical shift from looking at individual construction of the world that is atemporal, ahistorical, and acultural because it's by-and-large biologically articulated. [19]

Luis: Well perhaps I will start by mentioning something that I think is important to bear in mind when we discuss things of importance for activity theory. The idea of activity comes from VYGOTSKY. LEONT'EV in fact credits several passages from VYGOTSKY as emphasizing the role that activity plays in the way we come to think. Yet, what is very interesting is that while LEONT'EV is credited with having developed a theory of activity, this is not really accurate in the sense that LEONT'EV's goal was not to elaborate a theory of activity. Actually, within the structure in his seminal work "Activity, Consciousness and Personality" (LEONT'EV, 1978) he devotes only 12 pages to the structure of activity, to a discussion of what activity is about. The rest of the book is about something else, something that he considered to be the important thing. We have to remind ourselves often and often, again and again, that LEONT'EV was thinking as a psychologist. What he was trying to prove and to understand was how consciousness and personality emerge from activity. Not, as he says in the introduction of the book, in the sense that the human psyche is merely a derivative of activity, but rather how the human psyche arises out of the contradictions, the transformations, all the elements that come into play in activity—in short, how the psyche emerges from there. So this is why the focus of the book is really on consciousness and personality seen in terms of the activity in which individuals engage. He was led, at least, to describe certain elements and certain layers of activity, but not to theorize activity in general. Now, when we talk about activity theory, we talk rather in terms of developments that were made by ENGESTRÖM and other scholars. However, my feeling is that the central elements, this psychological dimension that LEONT'EV was talking about, have gotten lost and activity theory has become rather a very, very general way of describing what people do and the things that intervene or mediate activity. We have lost sight of this very penetrating and insightful idea of the relationship between the way we are as continuously changing beings and the activity we engage in. [20]

Michael: HOLZKAMP (1983) picks up on the importance of subjectivity in activity, on the role of the way the world appears to us in activity. HOLZKAMP develops Subjektwissenschaft, what in English what would be translated as "science of the subject." It's a science that looks at human beings and their consciousness all the while realizing that individual human consciousness is a concretization of possible consciousness that are available in the collective as a whole. HOLZKAMP also focuses on aspects that are important to other Marxist scholars in the sense that our life world—what is available to us, what is conscious to the individual or even to the group—cannot access directly from some of the structural determinations that come with institutional relations or the kind of cultural artifacts that we take on. Dorothy SMITH (e.g., 1990), for example, analyzes categories that we use in everyday talk, like single-parent family. The individual subject of activity no longer wonders about the work that concepts like these do because these words have been introduced into their worlds, affecting their interrelations. Being a single
mom or accepting this label leads to particular kinds of interactions with schools, for example, without people being aware of why it is that way, when in fact sociologists introduced the term single-parent family as a category into our common discourse and thereby affected the kind of discourses that we can have about social issues. HOLZKAMP is very similarly interested in individual subjectivity, but keeps alive this other dimension, that there are determinations that are outside of our consciousness that affect our ways of interaction so that the sources of the troubles that we experience may lie elsewhere, outside of our consciousness and yet affect our ways of interaction. So, for HOLZKAMP, learning is something very different from other people. Namely, it's an increase in the action possibilities of the individual as seen by the individual. And then learning is no longer problematic or motivation is not problematic because who would refuse being able to do more and better if it's part of the goal that they want to achieve? Who? You're inherently motivated. If learning means an expansion of my action possibility, I will inherently do it. His conception of learning becomes very different because he views it from the subject and as expansion of action possibility. What is interesting is that this line of thinking has influenced Jean LAVE. Jean LAVE knew Klaus HOLZKAMP but her citations of HOLZKAMP (e.g., in LAVE, 1993) have never been picked up on in the Anglo-Saxon literature. [21]

3.2 Developing theoretical tools to study mathematical thinking and learning

Luis: To continue with your question about what I think is essential in activity theory, activity theory has been developed in different directions and I think that there are two results that come out of that. On the one hand, I think that because activity tries to encompass many, many things, it is very, very generic as a theory. It becomes very generic, very general in order to encompass the diversity of activities that human being carry out in everyday life. This is a strength because you can apply activity theory to almost everything that human beings do. But on the other hand, it is it's weakness because when you try to apply it to a very particular research field you find yourself in a situation in which you can't find the theoretical tools that you need to tackle your specific research question. This happened to me. [22]

I started at the end of the 1990s videotaping in classrooms in a systematic way: by that time I had succeeded in framing my research questions in terms of a Vygotskian/Leont'evian perspective. Yet I found myself in need of new conceptual tools to understand and to make sense of what was going on in the classroom. I felt a need to develop new theoretical constructs that would allow me to respond to my research questions. It was at that juncture that the idea of objectification—the process of becoming consciously aware of a cultural conceptual object in activity—came out. It stems from the work of VYGOTSKY and the cognitive role that he tied to language. I remember that I was trying to understand the difficulties that students usually have in dealing with what we mean in mathematics by figure n in a sequence of figures. After a week of videotaping in a school, I had to go to Toronto. I brought with me "Mind in Society" (VYGOTSKY, 1978), that I had read several times. I was waiting for a taxi when I reached a
passage in which VYGOTSKY argues that children solve problems with the help of their speech, eyes, and hands. The picture of my students struggling with the meaning of figure n came to my mind, and I said "This is it!" But I was and remain unconvinced that internalization was the best way to cast the problem. And this is how the idea of objectification appeared to me as something more appropriate to understand my research questions and experimental data. Objectification is there in VYGOTSKY's work, but without really being articulated. The idea is present in LEONT'EV's work too, to a greater extent. Yet it is not really developed into something organically related to learning. That the idea of objectification is latent in VYGOTSKY's and LEONT'EV's work is not surprising as it is a central idea in the historical-dialectic tradition of HEGEL and MARX. It is curious that it was not really developed further. \[23\]

The development of the idea of objectification within the context of VYGOTSKY/LEONT'EV's perspective had its own challenges given, as Michael was saying, its emphasis on the historical and cultural dimensions of knowledge and knowing. This cultural-historical dimension was not a surprise to me, as I was able to perceive and to touch it, so to speak, through the historical mathematical documents that I studied in the early 1990s leading to publications around the history of algebra and algebraic thinking (e.g., RADFORD, 1995, 1997). So, bit-by-bit, it appeared to me that the problem was not just about finding theoretical tools to describe what the students are doing in a particular classroom activity, but of understanding this activity against the background of a cultural and historical setting as well. This is where the theory of objectification was developed. \[24\]

My first attempt to articulate the theory of objectification, in my Educational Studies in Mathematics article (RADFORD, 2000), was still overwhelmingly focused on language. It took me several years to incorporate the question of gestures and to introduce the idea of semiotic means of objectification in a more methodic manner (RADFORD, 2002, 2003). The first systematic exposition of the theory of objectification appeared in Spanish (RADFORD, 2006) with refinements in a 2008 chapter (RADFORD, 2008), where the idea of subjectification appeared with the idea of objectification. It is in this chapter that the question of ethics also came to the fore. So the relationship to what is usually called activity theory is clear. What I am interested in is not an activity in general. I am a mathematics educator, and my horizon of action is very specific: it is the school. What I am interested in is a particular kind of activity, the activity of teaching and learning, in particular teaching and learning of mathematics. So I had to create tools that evolved into a theory to account for the way in which students become acquainted, familiar with culturally and historically constituted forms of knowing and being. \[25\]

Michael: Just to add to the point that you—Luis—made earlier about activity theory being a very overarching, a very general framework—I noted the term heuristic. For me it's more of a heuristic for orienting myself to the world and thinking about what I need to be aware of. And so culture, history, and activity are things that I need to be aware of. But then, like you, I need to create tools that are
appropriate to the object of inquiry and this is a point that oftentimes doesn't
come into the discussions of method. We're talking about activity theory or CHAT
and we fall into a mistake that the creators of activity theory had not wanted us to
talk about activity in general and the abstract when, in fact, VYGOTSKY (e.g., 1989) wanted to have a "concrete human psychology." It's
about the real world, concrete situations we need to look at activities that we want
not only to understand but to transform. It is praxis that we want to understand
and transform. And so, that praxis really is at the heart of our endeavor, not the
theory for itself. It's a theory for action, it's not just a theory for understanding. It's
one that's there to assist us, perhaps, as a heuristic for going about transforming
this world, consistent with "The Eleventh Thesis on Feuerbach." It's about us human beings shaping our environment and creating tools that allow us to shape
it so that the world better provides for human needs. [26]

3.3 Dialectical thinking and implications for understanding activity

Lionel: Is an unspoken part of this a commitment to a dialectic ontology that
comes from HEGEL and MARX? Does cultural-historical activity theory all unfold
from the commitment to a dialectical perspective? [27]

Luis: Yeah, I think so. Dialectical thought is a way of expressing this fundamental
idea that life is something in motion, that we've got something really wrong with
our usual, common understanding of the verb to be. We are, but we are always
changing, so the phrase is always incomplete. When we say, "he is" or "I am," the
verb is always in transformation, and so is the subject! [28]

Michael: The dialectics is the aspect of activity theory that is the most difficult to
understand and the least attended to because inherent in dialectical
understanding is transformation—dialectics means or embodies inner
contradictions, not logical contradictions, inner contradictions. In LEONT'EV's
work you have the idea of reflection; consciousness reflects material reality. In
looking at any concrete activity, material reality and consciousness cannot be
separated. You therefore have a tension or dynamic there. These are two
aspects of the same thing: concrete activity. In activity theory you take a
dialectical perspective, which interestingly allows us to overcome any body-mind
separation, because in this dialectical concept we already begin thinking of body
and mind, mind and nature as an integrated unit in the sense that we cannot
understand one without the other. You can identify different aspects, but you
cannot understand them independently, in the same way you cannot understand
independently head and tail of a coin, nor the wave and corpuscle nature of light.
What is relevant, nature and objects or tools or artifacts, is a function of
consciousness. And consciousness, in turn, is not independent of the cultural-
historical material context within which it has evolved. [29]

There is another difficult aspect to understand—the relationship between thought
and affect. In VYGOTSKY's "Thought and Language" (1986) it is one of the
aspects least attended to. He thinks those two as inner reflections of material
reality, a point that LEONT'EV (1978) subsequently takes up and develops

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further. This is why thought and affect are thought to be part of a higher order unit—they are both reflections of or denote aspects of, but incompletely, of this higher unit. So it is only when we think of thought and affect as parts of one overarching whole that we can really understand thought/cognition because without it, thought would think itself. There's no reason for thought to be there. Thought is for a purpose and it's only affect that can bring this purpose in. [30]

Luis: Yes, I think this point relates to an essential difference between Vygotskian inspired approaches and traditional approaches that we see in mathematics education and elsewhere. The central idea that there is no separation between cognition and material reality is tremendously important. There is no separation between the individual and the society. There is no separation between I and other—yet, one cannot be reduced to the other. We have many, many approaches in math education and I would say that most of them, even if they don't acknowledge it, work within the framework of a dualistic view of the individual—the individual and the social. Sometimes the social appears as an antagonist entity. Sometimes the social appears just as an arena or a space where I find accommodations; I produce accommodations. But in the end all of these theories keep one way or another, at one layer or another, this idea of the separation between the individual and the social. Activity theory, in general, breaks with this tradition. It goes back to Benedict de SPINOZA (1989 [1677]) whose philosophy inspired VYGOTSKY a lot. This is why VYGOTSKY and Vygotskian inspired scholars recognize themselves as working within a monistic tradition as opposed to the Western dualistic tradition. [31]

3.4 Comparing activity theory with other research perspectives

Lionel: Comparing activity theory with other theoretical perspectives in education, is there anything that you'd like to add? Is there else that activity theory does that other theoretical perspectives don't do or, perhaps to put it another way, why does research in education need activity theory? [32]

Michael: It would be useful to put what we are talking about into the context of a study that we have done involving mathematics in an everyday world context. In one of our studies we looked at mathematics in a fish hatchery. Rather than just videotaping one of the workers explaining graphs and figures that they use, we looked at what they were doing, namely hatching fish. We looked at the particular mathematical forms or representations that they explained to us, as well as the emergence and the functions of these. For example, one of the fish culturists had a bimodal distribution of fish weight and a mono-modal distribution of lengths. And she tells us, while pointing to her graph, about how these are skinny and small fish, and these are short and sort of more bulky fish. You might say, if you have another approach, you might say something about her understanding of the graphs alone when, in fact, anything she knows about these distributions can be traced to events within the activity. I traced it through her work and how this understanding develops, as she takes one fish at a time, makes measurements, looks at the fish, and develops a sense for the kinds of things that go through her hands—literally—and the kinds of things that emerge from her hands on the
keyboard and a computer screen. So here are graphs and here she has fish. When she talks about the graphs, the fish are still present in her thinking and when she has the fish, the graphs are also present because she looks at the specimens and she can talk about where an individual might be located in the distribution. [33]

But the worker's use of particular forms of mathematical representation and the relationship of these to other forms of her works in itself, to me, was not sufficient for understanding her cognition because the events that I observed changed in the course of the five-year period while we were in the hatchery. Tools changed and with them the way these people acted. Sometimes they created knowledge and what they were doing changed as a result. Because I had just prior to that study become familiar with the historical dimensions of activity theory, our research team said, we cannot just look at what we have. We need to understand this working context as it has evolved historically. So while we studied this place empirically for five years, we had historical records of the fish hatchery for its 30-year existence, from newspapers, from paperwork that the fish hatchery has collected, and so on. [34]

But then we became aware that it's not just this fish hatchery. This particular fish hatchery is part of a larger system of hatching fish in British Columbia. This fish hatching system underwent historical changes. And, in part, this system exists because of this fish hatchery, because it's this fish hatchery that realizes it. This fish hatchery created knowledge that the system as a whole used in order to maintain itself. [35]

Part Two. It's not just that we have a federal government initiative to hatch fish in fish hatcheries. The fish hatcheries—they're connected tightly into local economy, that they both support and they are intended to support. So we have to study the local economy, the local society within which it functions. Salmon fishing provides work for fisherman. The thriving tourism in the area is a function of the salmon that is there to be fished. The four local aboriginal Indian bands live off of the salmon. We have to examine all of this because of activity theory. Because it told me, don't just look at it confined as if it was a box of events; attempt to understand the larger context. It is activity theory that enabled me to understand why I would be observing this kind of mathematics in this place at this time because I took into account all these other layers. [36]

Just to finish, because the question is about other theoretical perspectives. Other theoretical perspectives would have asked me to look at whatever the construction is in the head of this fish culturist—and that's it. Or it might have directed me to look at her discourse—discourse independent of the historical context, when in fact the discourse is, mathematical and otherwise, practical and so on, tied into the history of the system as a whole. CHAT doesn't tell me how to research it but it has made me aware that there is more than what is apparent to the eye if I hear a fish culturist talk about graphs. [37]
Luis: I think we can make exactly the same argument in school-based mathematics education research. In mathematics education, very often, the theoretical approaches that are used confine teaching and learning to what is going on in the classroom, making an abstraction of the social and the cultural and historical context that is beyond its walls. It is a common assumption amongst a large proportion of educational researchers that we can understand thinking and learning just by paying attention to the way the students are interacting. That has been my criticism of constructivism for many years. Constructivism has been very influential in education, and we certainly owe it a lot. However, constructivism has been one of those approaches in which everything is confined to the classroom. By not taking into consideration of all the different things such as beliefs, forms of understanding that students bring into the classroom nor the historicity of knowing and being constructivism fails to give us the big picture of the ways in which students and teachers are knowing and becoming in schools. [38]

Just to come back to Michael's example, there is also the participation of history in what he is saying, in the sense that you have this history that goes back to the moment in which this social institution—the fish hatchery—was created. But also, it goes back to forms of thinking that we pass on from generation to generation without being aware of it in a clear way. He was talking about graphs and the way fish were being related to abstract graphs. But the very idea that you can explain the growth of salmon, the weight and other aspects of the fish—within this Galilean way of thinking, the way that it makes sense to express these ideas of how the fish grow and evolve and so on, the way that it can be expressed mathematically, is part of a scientific tradition that we bring in when we teach mathematics, science, and other subjects. There is a ubiquitousness of the historical dimensions of practice that we are not really aware of when we teach science, mathematics, and other things. [39]

In one of my recent papers (RADFORD, 2009) I made an effort to show, for instance, something that we take for granted—that the idea of the origin in a Cartesian graph was the result of a very sophisticated process of disentangling ideas of space, of position, of distance that were in the beginning all put together. It took a long time, historically speaking, centuries to disentangle these ideas. It happened with DESCARTES, that these concepts were separated and he said that space, position, and distance were not the same thing. But in order for DESCARTES to talk about that, it was necessary that some historical and cultural conditions were available to him. When you talk about interpreting the growth of a fish or of a population of fish through a graph, there is a tremendous historical intelligence embedded in that. So when you think that knowledge will arise out of interaction, as face-to-face interaction alone, we are missing really the point. [40]

3.5 Mathematics and cultural-historical activity

Lionel: Where is mathematics in activity? You, Michael, talked about mathematics in the fish hatchery. Is it possible to frame mathematical activity on its own within
the activity of the fish hatchery? How do we sort out mathematics in activity within a context outside of or within the classroom? [41]

Michael: That's not what I tried to say or emphasize. It's not mathematical activity in a fish hatchery because fish hatching and the fish mathematics that we see, they're bound up with one another. It's not mathematics in an authentic context. It is something like fish hatchery math that is culturally and historically bound up, and economically bound up. And as Luis said, it's not just the 120 years of history that we studied—to contextualize the history of the fish hatchery, which contextualized what we observed over the five years we spent in the institution to contextualize individual learning events—but it's built on the cultural history which includes whatever work eventually led GALILEO and DESCARTES to conceptualize space as independent and sets up how we can make graphical representations that were difficult for GALILEO. GALILEO didn't have the nice graphs that we nowadays use to represent motion. [42]

Luis: Not even DESCARTES. DESCARTES made it possible for others to conceive of graphs in the manner we do nowadays, that is, graphs having two perpendicular metric axes—the so-called Cartesian Graphs. There were no Cartesian graphs in DESCARTES' work. [43]

Michael: See you—Lionel—say "mathematics," we have been talking about the mathematics, but this is a point I made in one piece about fish hatching (ROTH, 2005) and a point that I make in other places (e.g., ROTH, 1996) where people talk about context in mathematics—these are not the same things. A mathematician's mathematics, or children doing/seeking solutions to what happens when you have a piggy bank that has a dollar and you add two dollars per day, are concerned with something very different than the people in the fish hatchery and so I made a distinction. The fact that we use the word mathematics generalizes—it puts into the same bag—two or more things that are very different. In the first case, mathematics and mathematical objects are the objects of the activity. In the fish hatchery, it's not mathematical objects; they're but tools in order to raise fish. The fish are the object of the activity, not the mathematics. But the mathematics co-emerges with, an understanding of the mathematics co-emerges with, an understanding of the fish and that's what I have tried to articulate. The thing is that they understand their fish population by tracking them with these mathematical tools. But the understanding of the mathematical tools within this particular cultural-historical context only emerges because they already understand the fish. They are bound up with one another. The object of the activity for the fish culturist is not to understand statistics or the graph as graph—the properties of a graph. They want to raise fish; they want to have a million healthy 20-gram Coho. That's their purpose. This is different from the classroom data that Luis and I analyzed this morning—children in a 4th grade class who, at the end of their work, have something like $2n + 1$. In this case, thinking in an algebraic way is the object of activity, that's the outcome. That's what the teaching is set up to arrive at. In contrast, my fish culturist wants to have 20-gram fish that survive and make it in the ocean and come back in great numbers. That orientation to the ultimate outcome is very different and makes it a different
activity. That's what activity theory is about: understanding the object in the right way. So you cannot conflate the two mathematics. They're similar, there are similar things that appear, but they become very different in the concrete matters and in terms of the cognition or knowing that we observe. So the kids wrestle with the formulas and ways of representing numbers with the goal of satisfying the demands of their teacher, and the fish culturists that I've observed wrestle with raising healthy fish. [44]

Lionel: Anything to add Luis? [45]

Luis: Yes. I think that, depending on the activity, there is a shift of focus. In Michael's example, the focus is not the formula or the graph, but is using those ideas in order to make sense of and take actions as needed in order to reach the goal of the activity—the specific weight of the fish. This is why I was saying earlier that the kind of activity I am interested in is very, very specific. It is the teaching and learning of mathematics in a regular classroom. Activity within the classroom is oriented to give the students a chance to become acquainted with historical and cultural forms of mathematical thinking and being. The question is not about imposing a way of thinking, it is rather an invitation to think in ways that took centuries to achieve. Classroom activity, as I conceive it, is a process, as is learning. It is a process in which each student refracts differently a historically constituted form of thinking and being. This is why thinking about Cartesian graphs can be manifested in very different ways, sometimes as something more formal or symbolic, sometimes as something more kinesthetic, as the students that I discussed in my *International Journal on Mathematics Education* paper (RADFORD, 2009) taught me. And here we get into the question of the diversity of the manners in which we can think mathematically. I think that those individuals working in the fish hatchery were thinking mathematically too, but not perhaps in the savant way of thinking mathematically that we can find in a research lab or amongst professional mathematicians. Yet, in their activity, there is a relationship to number, there is a relationship to space, there is a relationship to time because you have to have your fish grow up in a certain span of time. So you have all those relationships and there is something intrinsically or profoundly mathematical about it. [46]

Something that is challenging for activity theorists, I think, is to try to understand this diversity of ways of thinking—in our case, ways of thinking mathematically. Our challenge is to try to avoid mathematics as something homogeneous, something that has to coincide with only one type of mathematics. If we do not develop this sensitivity to understand and appreciate other genuine forms of thinking mathematically, mathematics becomes oppressive; mathematics becomes an oppressive discourse. I prefer to think that through our classroom activities we submit an invitation to the students and that by accepting it, there is the promise that the students and we, that all of us, will become enriched. This is why I am very interested in this student-student interaction and student-teacher interaction, to try to understand patterns that can be more inviting than others. [47]
All this week, Michael and I have been watching and discussing some videos from my classroom research. We have been following students from grade four to grade five, trying to understand how forms of thinking and being become more and more refined, allowing the students to move more flexibly in the world of mathematics that we put there for them to act on (ROTH & RADFORD, 2011). So it's a form of empowerment. The students are not merely empowered because in grade five they can solve problems that were difficult in grade four. They are empowered because they have more tools to make critical assessments of what is going on around them. [48]

There is also a political decision that we have to face and be aware of that goes along the lines of your question about the differences between activity theory and other perspectives. When we compare activity theory approaches to other approaches, what does it come down to? One thing I would say is the question of subjectivity. We are understanding it in deeply different ways than from other classical approaches. You ask, why does research in education need activity theory at the present time? I would say we don't need it. If you want to go along the lines of individualistic thinking, you don't need it. You have very good theories already there. But if you want to understand subjectivity, knowing, and being in a different way, as something that links you to your cultural, historical, and social context, then those approaches may not get you there. I am not saying that activity theory approaches are the only way, but they do provide very powerful tools in order to conceptualize the individual in non-individualistic terms; individuals as being part of their community, as agents who interact with others all the time. Subjectivity as something that emerges from activity, and in activity, with activity. The other point of comparison is also this idea of knowing. Knowing is not about getting an option; it's not about constructing something. Knowing is about becoming positioned critically within a socially and historically and culturally constituted practice. [49]

3.6 Challenges and concerns working with activity theory

Lionel: Let’s talk about challenges and concerns with activity theory that you experience in your work. Who would like to begin? [50]

Michael: Luis and I, we have both been talking about historicity and temporality in learning and change. We’ve been talking about the fact that the language that we currently have available leads us to entities like: I am, you are, this is. All of these terms reify entities. They don't highlight process. Yet it's only because of process, of events, of interactions that students and teachers change. And so, by reifying entities we actually bar understandings to learning. So the big challenge for me is to work on, to contribute to research in which we come up with a new kind of language that orients us to process where learning, as Jean LAVE (1993) says, is not the problem, where identifying knowledge and reifying knowledge is the problem. Change is inevitable and it's there, but how do we articulate it without falling back into, you know, this and that? The teacher knows, the student knows little. So one of the challenges would be for me to think the zone of proximal development, for example, in terms of the interaction—which is symmetric—
without people being symmetrically located institutionally. If you focus on interaction, we have a dynamic. If that is our unit of analysis—the dynamic, then change is inherent. Developing a discourse without reifying things—entities, that’s for me a big challenge because I want to have a language that brings in, that is inherently, historical and inherently highlights the historical, the change, and the learning aspects. [51]

Luis: I share those worries and those challenges and I face those challenges a lot. But something that is of particular interest to me is the study of subjectification, because we are trying to move away from a substantialist view of the subject, that is, a view where the subject is conceptualized as identical to itself. In the equation I = I, the first I is a project, an opening towards the world; the second I is not identical to the first, as it is an answer that becomes in turn a new question, and so on. We have to come up with new ideas in order to describe this continuously unfolding process of becoming. [52]

Our natural language is full of traps. We have nouns that don't change. I've been called Luis since I was born and yet I have been always changing, changing as I've been moving from one place to another and doing some things and then other things. The name is the same but the person is not. And then one question is how do we characterize someone without using this is. This Matthew is ... Well Matthew, a Grade 4 student that I have been discussing with Michael over the past few days, is always changing but at the same time something that changes —remains, without which we wouldn't be able to pinpoint or identify Matthew. The problem is how to account for this sameness that is not. [53]

Michael: If you want to say that "Matthew learned," then you already have a discourse that forces you to say "Matthew 1," "Matthew 2," and "a change." But Matthew is changing, so the way in which our language functions is to use this substantialist discourse. So I would like to arrive at a dynamic perspective and I was thinking about Matthew and his engagement in mathematics. In the past what we have tended to say is that the student has a bad experience or a good experience in mathematics and then becomes interested or disinterested in mathematics. This is not a dynamic perspective; it's almost a causal reasoning and is global enough so you can say "Matthew—bad experience, drops out of math. Matthew—good experience, positive math identity." And this is, I think, the discourse we have to get away from and develop one in where we see a dynamic that "Matthew becomes discouraged in engaging with mathematical objects." But the same type of mathematical activity can bring him back and develop a sort of a positive stance. And, once we have a discourse that allows me to follow this, then my explanations are not simple anymore because there's not just Matthew, but there is a process. And, you know, cumulatively this process within an activity is one that is objectively trackable. We may find at one point when Matthew has to make a decision; he doesn't take calculus, for example, or he doesn't enroll in university mathematics. But it's not Matthew and a simple thing like his attitude. There are processes and I can follow Matthew over time, even at micro-levels where there are variations, continuous variations in the form and content of his engagement and the emotionality that is expressed and continuously changes.
So for me to say "Matthew 'is'" becomes very difficult because if it's anything, it's a process. And that's what I'd like to understand because that allows me to understand mathematical learning. [54]

Luis: Because we come back to this inseparability between knowing and being. [55]

Michael: Yeah. [56]

Lionel: Thank-you both. [57]

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